

University of Information Technology and Sciences
Faculty of Science and Engineering
Department of Computer Science and Engineering
Program: B.Sc. in CSE
Term Final Examination, Autumn-2022
Course Title: Differential and Integral Calculus
Course Code: MAT 163

Marks: 50

Time: 3(three) hours

(Answer all questions)

1. a) What is limit? Evaluate the following limits. [5]
By using L'Hospital rule evaluate:
(i) $\lim_{x \rightarrow 0} (e^x + x)^{1/x}$ (ii) $\lim_{x \rightarrow \pi/4} (1 - \tan x) \sec 2x$
- b) (i) If $f(x, y, z) = x^3 y^5 z^7 + xy^2 + y^3 z$, then find $f_x, f_y, f_z, f_{xz}, f_{yyy}$. [5]
(ii) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = x^4 \sin(xy^3)$.
2. a) Find the intervals on which the function $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$ is increasing and decreasing. [5]
- b) State Extreme-Value theorem. Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 15x^2 + 36x$ on the interval $[1, 5]$, and determine where these values occur. [5]
3. Evaluate the indefinite integrals: [10]
(i) $\int (x^{-3} - 3x^{1/4} + 8x^2) dx$ (ii) $\int x^2 \sqrt{x-1} dx$ (iii) $\int x \ln x dx$
(iv) $\int \frac{\sqrt{x^2-9} dx}{x}$; assuming that $x \geq 3$
4. a) Evaluate the definite integrals : [5]

$$(i) \int_{\sqrt{2}}^2 \frac{dx}{x^2 \sqrt{x^2 - 1}} \quad (ii) \int_0^1 \tan^{-1} x \, dx$$

b) By using gamma and beta function evaluate the following integrals.

[5]

$$(i) \int_0^1 x^6 (1 - x^2)^{1/2} dx$$

$$(ii) \int_0^{\pi/2} \sin^8 \theta \cos^6 \theta \, d\theta$$

5. a) Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$.

[4]

b) Evaluate the following multiple integrals:

[6]

$$(i) \int_0^{\ln 3} \int_0^{\ln 2} e^{x+y} \, dy \, dx$$

$$(ii) \int_0^1 \int_0^y \int_0^{\sqrt{1-y^2}} z \, dz \, dy \, dx$$