University of Information Technology & Sciences (UITS)
Faculty of Science and Engineering
Department of Computer Science and Engineering
Program of B.Sc. in CSE
Mid Term Examination, Autumn- 2023
Course Title: Differential and Integral Calculus
Course Code: MAT 163

Marks: 20   Time: 1(one) hour

(Answer all questions)

1. (a) Find domain and range of the following functions and also sketch the graph of the following functions : **[03]**

$$(i) f(x) = \sqrt{(-x-2)} - 4 \quad (ii) f(x) = \frac{x}{x+3}$$

  (b) Define even function and odd function. Test whether the following functions are even or odd. **[04]**

$$(i) \ f(x) = (x + \sqrt{1 + x^2})$$

$$(ii) \ f(x) = \frac{tanx}{x+sinx}$$

  (c) If $f(x) = \sqrt{x^3 + 2\sqrt{x}}$, $g(x) = (1+x)^{-1}$ and $h(x) = x^{3/2}$. **[03]**

find $(f \circ g \circ h)(x)$.

2. (a) A function $f(x)$ is defined as follows. **[02]**

$$f(x) = \begin{cases} 2x + 3, & x \le 4 \\ 7 + \dfrac{16}{x} & x > 4 \end{cases}$$

Discuss the continuity of $f(x)$ at $x = 4$.

  (b) Find $\frac{dy}{dx}$. **[08]**

(i) $x^3y + 4xy^2 = 3xy$ (ii) $sin(x^2y^2) = x$

(iii) $y = \frac{sinx}{1+cosx}$ (iv) $y = [1 + cos^3(sin2x)]^{-3}$

University of Information Technology and Sciences
Faculty of Science and Engineering
Department of CSE
Term Final Examination, Autumn- 2023
Course Title: Differential and Integral Calculus
Course Code: MAT 163

Marks: 50   **Time: 3 hours**

(Answer all the questions)

1. **a)** Evaluate the following limits. **[6]**

     (i) $\lim_{y \to \infty}(\sqrt{y^6 + 5y^3} - y^3)$ (ii) $\lim_{x \to 0}\left(\frac{1}{x} - \frac{1}{sinx}\right)$ (iii) $\lim_{n \to 0}(1+x)^{1/x}$

   b) (i) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = x^4 \sin(xy^3)$. **[4]**

       (ii) Find the second order partial derivatives of $z = x^2y^3 + x^4y$.

2. **a)** State the Mean Value theorem. Verify the Mean value theorem for **[5]**

$$f(x) = x^3 + x - 4; [-1, 2].$$

   b) State Extreme-Value theorem. Find the absolute maximum and minimum values **[5]**
of the function $f(x) = 2x^3 + 3x^2 - 12x$ on the interval [-3, 2], and determine
where these values occur.

3. Evaluate the indefinite integrals: **[10]**

     (i) $\int(x^{-3} - 3x^{1/4} + 8x^2)\, dx$ (ii) $\int x^2\sqrt{x-1}\, dx$ (iii) $\int x^2 e^{-2x}\, dx$
     (iv) $\int \frac{\sqrt{x^2-9}}{x}\, dx$; assuming that $x \geq 3$.

4. **a)** Evaluate the definite integrals : **[6]**

     (i) $\int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}}$ (ii) $\int_0^1 \tan^{-1}x\, dx$ (iii) $\int_0^{\sqrt{\pi/2}} 5x\cos(x^2)\, dx$

   b) By using gamma and beta function evaluate the following integrals. **[4]**

     (i) $\int_0^{\pi/2} \sin^5\theta \cos^6\theta\, d\theta$ (ii) $\int_0^1 y^{3/2}(1-y)^{5/2}\, dy$

5. **a)** Evaluate : (i) $\int_0^1 \int_{-3}^2 y^2 x \, dy \, dx$ (ii) $\int_0^{\pi/3} \int_0^{\cos y} x \sin y \, dx dy$ [4]

    b) Evaluate the triple integral $\iiint_G 12xy^2 z^3 \, dV$ over the rectangular box G defined by the inequalities $-1 \le x \le 2, 0 \le y \le 3, 0 \le z \le 2$. [3]

    c) Find the area of the region that is enclosed between the curves [3]

$$y = x^2 \text{ and } y = x + 6.$$