Department of Computer Science and Engineering

Program: B.Sc. in CSE

Term Final Examination, Spring-2024

Course Title: Differential and Integral Calculus

Course Code: MAT 0541111

Marks: 50 *Time: 3(three) Hours*

(Answer all the questions)

1. a) Evaluate the following limits: [06]

(i) $\lim_{s\to\infty}\sqrt[3]{\frac{2+3s-5s^2}{1+s^8}}$ (ii) $\lim_{x\to 0}(e^x+x)^{\frac{1}{x}}$ (iii) $\lim_{x\to 0}(\frac{1}{x}-\frac{1}{sinx})$

b) (i) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = e^{xy}\sin(4y^2)$ [04]

(ii) Find $\frac{\partial^2 z}{\partial x^2}$ and $\frac{\partial^2 z}{\partial y^2}$ if $z = \sqrt{x}\,cosy$.

2. a) State Rolle's theorem. Verify Rolle's theorem for the function [05]

$$f(x) = x^3 - 3x^2 + 2x, \quad x \in [0,2].$$

b) State Extreme-Value theorem. Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 15x^2 + 36x$ on the interval [1, 5] and determine where these values occur. [05]

3. Evaluate the following integrals: [10]

(i) $\int x^2\sqrt{1+x}\,dx$ (ii) $\int\frac{\sqrt{x^2-25}}{x}dx$; assuming $x \geq 5$ (iii) $\int\frac{\cos\sqrt{y}}{\sqrt{y}}\,dy$

(iv) $\int\frac{e^x}{\sqrt{1-e^{2x}}}dx$ (v) $\int xe^{-2x}dx$

4. a) Evaluate the definite integrals : [04]

(i) $\int_4^9\left(4y^{-1/2}+2y^{1/2}+y^{-5/2}\right)dy$ (ii) $\int_0^1\tan^{-1}x\,dx$

b) Evaluate the following integrals by using Beta & Gamma function: [06]

(i) $\int_0^{\pi/2}sin^5xcos^4\,xdx$ (ii) $\int_0^{\pi/2}sin^{12}x\,dx$ (iii) $\int_0^1 x^4(1-x)^{\frac{3}{2}}dx$

5. a) Evaluate : (i) $\int_0^1\int_{-x}^{x^2} y^2x\;dy\,dx$ (ii) $\int_0^{\pi/3}\int_0^{cosy} xsiny\;dx\,dy$ [06]

(iii) $\int_{-1}^2\int_0^3\int_0^2 12xy^2z^3dzdydx$

b) Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$. [04]