

Program: B. Sc. in CSE and Engineering
Term Final Examination, Autumn-2024
Course Title: Differential and Integral Calculus
Course Code: MATH 0541111

Marks: 50

Time: 3 (three) hours

(Answer all the questions)

Q. No.	Questions	Marks
1. a)	Evaluate the following limits: (i) $\lim_{x \rightarrow +\infty} (\sqrt{x^6 + 5x^3} - x^3)$ (ii) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$ (iii) $\lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}}$	[6]
b)	(i) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = y^{-3/2} \tan^{-1}(x/y)$ (ii) Find the second order partial derivatives of $z = e^x \cos y$.	[4]
2. a)	Find the intervals on which $f(x) = 2x^3 + 3x^2 - 36x$ is increasing and the intervals on which it is decreasing.	[5]
b)	State Extreme-Value theorem. Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 15x^2 + 36x$ on the interval $[1, 5]$ and determine where these values occur.	[5]
3.	Evaluate the following integrals: (i) $\int x^2 \sqrt{x-1} dx$ (ii) $\int \frac{\sqrt{x^2-16}}{x} dx$; assuming $x \geq 4$ (iii) $\int y^4 \sqrt[3]{5-4y^5} dy$ (iv) $\int \frac{5x-10}{x^2-3x-4} dx$ (v) $\int x^2 \ln x dx$	[10]
4. a)	Evaluate the definite integrals : (i) $\int_0^{\ln 2} e^y (1 + e^x)^{\frac{1}{2}} dy$ (ii) $\int_1^{\sqrt{2}} \frac{1}{x^2 \sqrt{4-x^2}} dx$	[4]
b)	Define Beta and Gamma Functions. Evaluate the following integrals by using Beta and Gamma function: (i) $\int_0^{\pi/2} \sin^5 x \cos^4 x dx$ (ii) $\int_0^1 x^5 (1-x)^{5/2} dx$	[6]
5. a)	Evaluate : (i) $\int_0^{\pi/3} \int_0^{\cos y} x \sin y dx dy$ (ii) $\int_{-1}^2 \int_0^3 \int_0^2 12xy^2z^3 dz dy dx$	[6]
b)	Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$.	[4]

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