Mid Term Examination, Spring 2024
Course Title: Discrete Mathematics
Course Code: CSE 153

University of Information Technology & Sciences (UITS)
Department of Computer Science and Engineering
Program: B.Sc. in CSE
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Marks: 20 **Time: 1 (One) hour**

[Answer all Five(5) question sets. And, from each question set, answer only one question X where X = MOD(ID, max_option) . So, if X=0 then "a", if X=1 then "b", if X=2 then "c" and so on. ID is last three digit of your student ID]

1. Let p and q be the propositions p: "I bought a lottery ticket this week." q: "I won the million dollar jackpot". Express each of these propositions as an English sentence. **[04]**
   a) $\neg p$ b) $p \vee q$ c) $p \rightarrow q$ d) $p \wedge q$
   e) $p \leftrightarrow q$ f) $\neg p \rightarrow \neg q$ g) $\neg p \wedge \neg q$ h) $\neg p \vee (p \wedge q)$.

2. "You can see the movie only if you are over 18 years old or you have the permission of a parent". **[04]**
   Express your answer in terms of
   m: "You can see the movie,"
   e: "You are over 18 years old," and
   p: "You have the permission of a parent."

3. Let N(x) be the statement "x has visited Noakhali," where the domain consists of the students in your school. Express each of these quantifications in English. a) $\exists x N(x)$ b) $\forall x N(x)$ c) $\neg\exists x N(x)$ d) $\exists x \neg N(x)$ e) $\neg\forall x N(x)$ f) $\forall x \neg N(x)$ **[04]**

4. Express these system specifications using the propositions p: "The user enters a valid password," q: "Access is granted," and r: "The user has paid the subscription fee" and logical connectives (including negations). **[04]**
   a) "The user has paid the subscription fee, but does not enter a valid password."
   b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."
   c) "Access is denied if the user has not paid the subscription fee."
   d) "If the user has not entered a valid password but has paid the subscription fee, then access is granted."

5. Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables. **[04]**
a) Someone in your school has visited Uzbekistan.
b) Everyone in your class has studied calculus and C++.
c) No one in your school owns both a bicycle and a motorcycle.
d) There is a person in your school who is not happy.
e) Everyone in your school was born in the twentieth century.

Bonus

Let F(x, y) be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody can fool Fred.
b) Evelyn can fool everybody.
c) Everybody can fool somebody.
d) There is no one who can fool everybody.
e) Everyone can be fooled by somebody.
f) No one can fool both Fred and Jerry.
g) Nancy can fool exactly two people.
h) There is exactly one person whom everybody can fool.
i) No one can fool himself or herself.
j) There is someone who can fool exactly one person besides himself or herself.

$\forall x\ F(x, y)$

$\exists x\ \forall y\ \neg F(x, y)$

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$\forall x F(x, y)$

Marks: 50 **Time: 3*(three)* hours**

(Answer all questions)

$\forall x F(x, y)$

1. Let F (x, y) be the statement "x can fool y," where the domain consists of all people in the world. Let Q(x, y) be the statement "x has sent an e-mail message to y," where the domain for both x and y consists of all students in your class.

a) Interpret each of these statements into a logical statement for F(x,y): **[06]**

1) Everybody can fool Fred. $\forall x F(x, y)$
2) Evelyn can fool everybody.
3) Everybody can fool somebody

b) Identify each of these quantifications in English for Q(x,y): **[04]**

1) $\exists x \exists y Q(x, y)$
2) $\exists x \forall y Q(x, y)$

2. Passage A: "It is below freezing and raining now. Therefore, it is below freezing now."

Passage B: "A student in this class has not read the book," and "Everyone in this class passed the first exam."

a) Demonstrate which rule of inference is the basis of the argument in passage A **[02]**

b) Explain Modus ponens and Modus tollens rules. **[02]**

c) Implement the conclusion "Someone who passed the first exam has not read the book." from passage B **[06]**

3. Proof by contrapositive and proof by contradiction are both indirect proof techniques in logic, but they take slightly different approaches:
Proof by Contrapositive relies on the fact that an implication statement (if P then Q) is logically equivalent to its contrapositive (if not Q then not P). Proof by Contradiction assumes the opposite of what you want to prove and then shows that this assumption leads to a logical contradiction.

a) Evaluate using contrapositive for an integer n, n is even if and only if n^2 is even. **[05]**

b) Apply the contradiction rule to prove that if n is an integer and n^3+5 is odd, then n is even. **[05]**

$p \to q$ $\forall x\ P(x)$ $n^3+5 = 2n+1$ $(2k^3+5)$ $8k^3+5$ 2^3+5 $(2k+1)^3+5$

4. Let A and B be sets. The Cartesian product of A and B, denoted by A x B, is the set containing all ordered pairs (a, b) where a is an element in A and b is an element in order b ∈ B.Let A = {a, b, c}, B = {x, y}, and C = {0, 1}.

a) Recognize the set operation and find **[06]**
- A × B × C.
- C × B × A.

b) For a single set M, identify M^2 if **[04]**
- M = {0, 1, 3}.
- M = {1, 2, a, b}.

5. In discrete mathematics, a relation describes a connection between elements in one or more sets. Floor Function (denoted by ⌊x⌋ or floor(x)), takes a real number x as input. Outputs the greatest integer that is less than or equal to x.

a) Analyze the relation R is an equivalence relation or not for the set A = { 1, 2, 3, 4, 5 } given by the relation R = { (a, b):|a-b| is even }. **[04]**

b) Demonstrate that floor(2x)=floor(x)+floor(x+1/2) in two cases when **[06]**
case 1) 0<=∈<½ and
case 2) ½<=∈<1

R ⊆ A

2, 4, 6, 8, 10